Advanced Microeconomics (II)

Based on the lectures of

Pro. Andrea Prat and Hanming Fang

Lecture 2: Auctions

Jehle-Reny

Ancient mechanism

New applications: spectrum auctions, online auctions, etc.

Paul Klemperer, Auctions: Theory and Practice (Available on: http://www.paulklemperer.org/)

Vijay Krishna, *Auction Theory*, 2002.

The traditional formats:

- First-price sealed bid;
- Second-price sealed bid;
- English auction
- Dutch auction

What is an Auction?

- **auc•tion**
  - A public sale of property or merchandise to the highest bidder.
  - A market institution with explicit rules which determine prices and the allocation of resources based on bids.
  - Bidding in the game of bridge.

- Derivation: From the Latin “auctus”, which is the past participle of “augere”, to increase.

Auctions

- *"The Greatest Auction in History"*
  - *Safire, William. "The Greatest Auction Ever: Get Top Dollar For the Spectrum,"The New York Times, 16 March 1995.*

Types of Rules: Open Outcry

- *1.English auction*. Price increases until only one bidder remains
- *2.Dutch auction*. Price decreases until some bidder jumps in

Types of Rules: Sealed Bid

- *1.First-price*. Winner pays its own bid. Losers pay nothing.

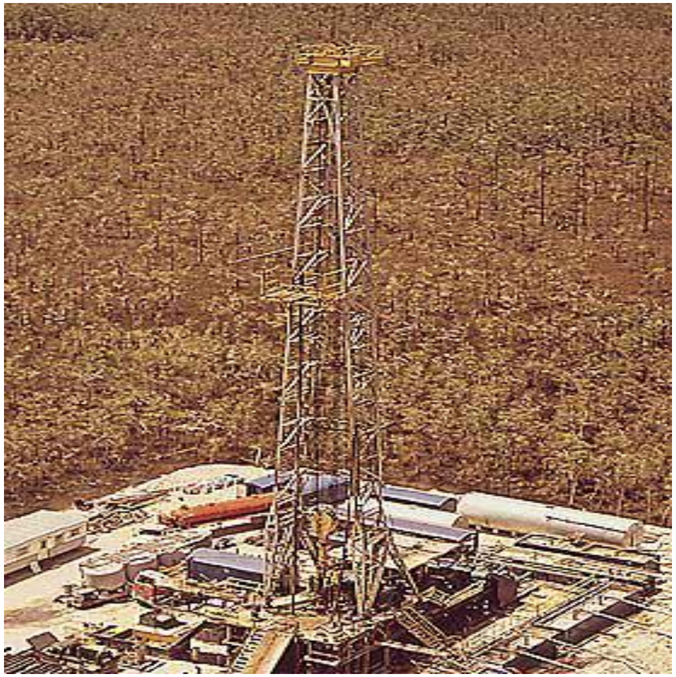
- *2.Second-price*. Winner pays highest losing bid. Losers pay nothing.

Types of bidders: Private Value

- Food
- Your valuation does not depend on others

Type of bidders: Common Value

- Unproven field
- Object has *same* value to all bidders, but each only has an *estimate* of that common value



“ Auctions in Disguise”

- Many interactions have the hallmarks of an auction:
  - There is a **prize**
  - Prize has **value**
  - Each party makes a **bid**where highest bidder gets prize
  - Bidding has a **cost**, where higher bids don’t cost less

Hiring Decision

- McKinsey and Charles River are trying to recruit Sven
- Whoever makes the highest wage offer will get Sven
- What type of bidders?
- What type of rules?

Labor Dispute

- Labor and management have a dispute over new work rules
- Work stops until some side gives in
- What type of bidders?
- What type of rules?

Promotion Tournament

- Amande and Mertare contenders to become the firm’s next CEO
- Whoever spends the most weekends in the office gets the job
- What type of bidders?
- What type of rules?

Competitive Negotiation

- Boeing and Airbus are each trying to get Iberia's business
- Iberia's CFO forces the two firms to continue beating each other's best offers and counteroffers until someone gives up
- What type of bidders?
- What type of rules?

Digression: Order Statistics

- Let $\tilde{\mathbf{v}} \equiv (\tilde{v}_1, ..., \tilde{v}_n)$ be a random vector, with $\tilde{v}_i \sim F(v)$ with continuous and positive density function $f(v)$ and support $[0, 1]$.
- $\tilde{v}_i \sim F(v)$ means that

$$\Pr(\tilde{v}_i \le v) = \Pr(\tilde{v}_i < v) = F(v).$$

- The first order statistics of n independent random variables $(\tilde{v}_1, ..., \tilde{v}_n)$ is denoted by $\tilde{v}_{(1)}$. The CDF of $\tilde{v}_{(1)}$ is

$$F_{(1)}(v) = \Pr\left(\tilde{v}_{(1)} \le v\right) = \Pi_{i=1}^n \Pr(\tilde{v}_i \le v) = F(v)^n$$

- Graph of how $F_{(1)}(v)$ changes with n.
- The PDF of $\tilde{v}_{(1)}$ is thus

$$f_{(1)}(v) = nF(v)^{n-1} f(v).$$

Digression: Order Statistics

- The second order statistics of n independent random variables $(\tilde{v}_1, ..., \tilde{v}_n)$ is denoted by $\tilde{v}_{(2)}$. The CDF of $\tilde{v}_{(2)}$ is

$$
\begin{aligned}
F_{(2)}(v) &= \Pr\left(\tilde{v}_{(2)} \leq v\right) \\
&= \Pr\left(v_i \leq v \text{ for all } i\right) \\
&\quad + \Pr\left(\left(\tilde{v}_i > v, \tilde{v}_j \leq v \text{ for all } j \neq i\right), \text{ for } i = 1, ..., n\right) \\
&= F(v)^n + nF(v)^{n-1}\left[1 - F(v)\right] \\
&= F(v)^n + n\left[F(v)^{n-1} - F(v)^n\right]
\end{aligned}
$$

- The PDF of $\tilde{v}_{(2)}$ is thus

$$
\begin{aligned}
f_{(2)}(v) &= nF(v)^{n-1}f(v) + n\left[(n-1)F(v)^{n-2} - nF(v)^{n-1}\right]f(v) \\
&= n(n-1)F(v)^{n-2}\left[1 - F(v)\right]f(v)
\end{aligned}
$$

Digression: Order Statistics

- It is also useful to denote a random variable

$$\tilde{y} = \max\{\tilde{v}_2, ..., \tilde{v}_n\}$$

as the max of $n-1$ independent drawings from CDF $F(\cdot)$.

- The random variable $\tilde{y}$ is useful because to bidder 1, $\tilde{y}$ is the max of the values of the other bidders.
- Let $G(y)$ denote the CDF of $\tilde{y}$:

$$\begin{aligned} G(y) &= F(y)^{n-1} \\ g(y) &= (n-1) F(y)^{n-1} f(y). \end{aligned}$$

Uniform Example

- $F(v) = v$ and $f(v) = 1$ for $v \in [0, 1]$;
- $E[\tilde{v}_i] = \frac{1}{2}$;
- The first order statistics of a sample of size n, $\tilde{v}_{(1)}$, has

$$\begin{aligned} F_{(1)}(v) &= v^n \\ f_{(1)}(v) &= nv^{n-1} \end{aligned}$$

and

$$E\tilde{v}_{(1)} = \int_0^1 v \left[nv^{n-1}\right] dv = \frac{n}{n+1} v^{n+1} \Big|_0^1 = \frac{n}{n+1},$$

which is increasing in n and approaches 1 as $n \to \infty$.

Uniform Example

- The second order statistics of a sample of size n, $\tilde{v}_{(2)}$, has

$$
\begin{aligned}
f_{(2)}(v) &= n(n-1)F(v)^{n-2}[1-F(v)]f(v) \\
&= n(n-1)v^{n-2}(1-v) \\
E\tilde{v}_{(2)} &= \int_0^1 v\left[n(n-1)v^{n-2}(1-v)\right]dv \\
&= n(n-1)\int_0^1 \left(v^{n-1}-v^n\right)dv \\
&= n(n-1)\left[\frac{v^n}{n}-\frac{v^{n+1}}{n+1}\bigg|_0^1\right] \\
&= n(n-1)\left(\frac{1}{n}-\frac{1}{n+1}\right)=\frac{n-1}{n+1}.
\end{aligned}
$$

- Because we earlier showed that the seller's expected revenue from SPA is:

$$R^{SPA}=E\tilde{v}_{(2)}=\frac{n-1}{n+1}.$$

Model

1 seller (auctioneer), N buyers (bidders)

The seller has one good which has zero value to her.

Bidder i has valuation $v_i \in [0, 1]$, where v_i has CDF F_i and density f_i.

Key assumption: *private values*: the v_i are independent across bidders.

One could have assumed common values.

The density functions are common knowledge.

Let the sale price be p. If i wins the object he gets payoff $v_i - p$. If he loses, he gets zero.

1. First Price

For the first-price auction, assume that $f_1 = ... = f_N = f$.

Format:

- Each i submits $b_i \geq 0$.

- The player with the highest b_i gets the object and pays b_i.

Strategy: bidding function $b_i(v_i)$.

Assume there exists an equilibrium where:

1. For every agent, $b_i(\cdot)$ is the same. Call it $\hat{b}(\cdot)$

2. The bid function $\hat{b}(\cdot)$ is strictly increasing

In such an equilibrium:

- Bidder i wins iff $\hat{b}(v_i) > \hat{b}\left(v_j\right)$ for all $j \neq i$ (disregard ties, as they happen with zero probability)

- $\hat{b}(v_i) > \hat{b}\left(v_j\right)$ iff $v_i > v_j$.

- A deviation by i is $b_i \neq \hat{b}(v_i)$ and we can find $\hat{b}(r) = b_i$.

- We can think of a deviation as "pretending to be type r rather than type v."

- The probability that i wins the object in a deviation is

$$\text{Pr}\left(v_j < r \text{ for all } j \neq i\right) = F(r)^{N-1}$$

Payoff of a bidder with valuation v who behaves as bidder with valuation r:

$$u(r, v) = (F(r))^{N-1}\left(v - \hat{b}(r)\right),$$

No deviation is profitable if

$$u(v, v,) = \max_r (r, v),$$

which yields the necessary first-order condition:

$$\left.\frac{\partial}{\partial r} u(r, v)\right|_{r=v} = 0.$$

or

$$\begin{aligned} & (N-1)(F(v))^{N-2} f(v)\,\hat{b}(v) + (F(v))^{N-1}\,\hat{b}'(v) \\ = \; & (N-1)\, v f(v)\,(F(v))^{N-2}. \end{aligned}$$

Left hand side: marginal cost. Right hand side: marginal benefit.

The left hand-side can be re-written as

$$\frac{d}{dv}\left((F(v))^{N-1}\,\hat{b}(v)\right)$$

Solve the differential equation:

$$(F(v))^{N-1}\,\hat{b}(v) = (N-1)\int_0^v x f(x)\,(F(x))^{N-2}\,dx + k.$$

But $k = 0$ because $\hat{b}(0) = 0$.

$$\begin{aligned}\hat{b}(v) &= \frac{N-1}{(F(v))^{N-1}} \int_0^v x f(x) (F(x))^{N-2} \, dx \\ &= \frac{1}{(F(v))^{N-1}} \int_0^v x d\left((F(x))^{N-1}\right).\end{aligned}$$

As we assumed, the bid function is strictly increasing in v.

(This is not a complete proof because we only looked a first-order conditions).

Proposition 4 *The first-price auction has a symmetric equilibrium in which*

$$\hat{b}(v) = \frac{1}{(F(v))^{N-1}} \int_0^v x d\left((F(x))^{N-1}\right).$$

Bidder i bids the expected valuation of the second-highest bidder conditional on i having the highest valuation.

Example: Suppose v is uniformly distributed: $F(v) = v$.

$$\begin{aligned}
\hat{b}(v) &= \frac{1}{v^{N-1}} \int_0^v x dx^{N-1} \\
&= \frac{N-1}{v^{N-1}} \int_0^v x^{N-1} dx \\
&= \frac{N-1}{Nv^{N-1}} v^N \\
&= \frac{N-1}{N} v.
\end{aligned}$$

1. Bidders *shade* their bid.

2. Shading decreases as N goes up.

2. Dutch Auction

The price p starts at 1 and decreases continuously over time.

When a bidders says “stop”, the auction ends. That bidder gets the object and pays price p.

A strategy is a price $p_i(v_i)$ at which the bidder says “stop”.

This game is strategically equivalent to the first-price auction.

Proposition 1 applies.

3. Second-Price Auction

Go back to the general model with f_i.

Format:

- Each i submits $b_i \geq 0$.
- The player with the highest b_i gets the object and pays the second-highest bid.

Proposition 5 $b_i(v_i) = v_i$ *is the unique weakly dominant bidding strategy for each bidder.*

Proof. Consider bidder i. Let b_{-i} be the vector of bids of the other players and B the highest bid by another player.
Player i gets payoff

$$u_i(b_i, B) = \begin{cases} 0 & \text{if } b_i < B \\ v_i - B & \text{if } b_i > B \end{cases}.$$

He prefers $v_i - B$ if $v_i > B$.

By bidding $b_i = v_i$, he ensures that he always gets the maximum payoff: 0 if $b_i = v_i < B$, and $v_i - B$ if $b_i = v_i > B$.

Therefore $b_i(v_i) = v_i$ is a weakly dominant strategy.

To check that it is the only weakly dominant strategy, suppose i uses a strategy $\tilde{b}_i$ that is different from $b_i(v_i) = v_i$ for some v_i.

If $\tilde{b}_i(v_i) > v_i$, we can find $B \in \left(v_i, \tilde{b}_i(v_i)\right)$ such that the payoff is negative, while it is zero with $b_i(v_i) = v_i$.

If $\tilde{b}_i(v_i) < v_i$, we can find $B \in \left(\tilde{b}_i(v_i), v_i\right)$ such that the payoff is zero, while it is strictly positive with $b_i(v_i) = v_i$. ■

4. English Auction

Format (slightly different from Sotheby's):

- The price p increases continuously over time.

- Bidder i can drop out of the auction anytime.

- When the second-last player drops, the last player wins the object and pays the price at the moment in which the second-last dropped out.

The parallel English-Second is not as straightforward as Dutch-First.

The two formats are not strategically equivalent. In the English Auction, player i learns that some players have dropped. This gives him information about their type.

But...

Proposition 6 *The unique weakly dominant strategy is to drop out when $p = v_i$.*

Proof. Suppose i drops out when $p < v_i$. If all remaining players drop out before p reaches v_i, he gets a strictly lower payoff than the one he would have gotten if he waited.

Suppose i does not stay on after p reaches v_i. If all remaining players drop out, he receives a negative payoff. ■

Revenue Comparison

First $=$ Dutch

Second $=$ English

First $\lesseqgtr$ Second?

1. *Revenue in First Price*

$$R_1 = \int_0^1 \hat{b}(v)\, g(v)\, dv,$$

where $g(v)$ is the density function of $v^{\max} = \max\{v_1, ..., v_N\}$. As

$$\Pr(v^{\max} \le v) = (F(v))^N,$$

we have

$$g(v) = \frac{d}{dv}(F(v))^N = N f(v)(F(v))^{N-1}.$$

Hence

$$\begin{aligned}
& R_1 \\
= \; & N \int_0^1 \frac{1}{(F(v))^{N-1}} \left(\int_0^v x d\left((F(x))^{N-1}\right)\right) f(v)(F(v))^{N-1} d \\
= \; & N \int_0^1 \int_0^v x d\left((F(x))^{N-1}\right) f(v)\, dv \\
= \; & N(N-1) \int_0^1 \int_0^v x f(x)(F(x))^{N-2} f(v)\, dx dv \\
= \; & N(N-1) \int_0^1 \int_x^1 x f(x)(F(x))^{N-2} f(v)\, dv dx \\
= \; & N(N-1) \int_0^1 x f(x)(F(x))^{N-2}(1 - F(x))\, dx.
\end{aligned}$$

2. *Revenue in Second Price*

$$R_2 = \int_0^1 \hat{b}(v)\, h(v)\, dv,$$

where $h(v)$ is the density function of the second-highest element v^{second} of $\{v_1, ..., v_N\}$. Note that

$$\Pr\left(v^{(2)} \leq v\right) = (F(v))^N + N(F(v))^{N-1}(1 - F(v)),$$

and

$$h(v) = N(N-1)(F(v))^{N-2} f(v)(1 - F(v)).$$

Hence,

$$R_2 = N(N-1)\int_0^1 v\,(F(v))^{N-2} f(v)(1 - F(v))\, dv.$$

We see that $R_1 = R_2$.

Proposition 7 *The auctioneer's expected revenue is the same in the four auction formats.*

Coincidence?

The Revenue Equivalence Theorem

Direct selling mechanism:

- Probability assignment functions: chance that i gets the object given a vector of reported values

$$p_1(v_1, .., v_N), ..., p_N(v_1, .., v_N),$$

such that $\sum_i p_i \leq 1$ (the auctioneer could keep the object).

- Cost functions: cost paid by bidder i given a vector of reported values (he may pay even if he does not get the object):

$$c_1(v_1, .., v_N), ..., c_N(v_1, .., v_N).$$

The cost could be negative, ie the auctioneer pays the bidder.

The equilibria we have considered in the four formats have corresponding equilibria in direct selling mechanisms.

1. First-Price, Dutch:

$$p_i(v_1, .., v_N) = \left\{ \begin{array}{ll} 1 & \text{if } v_i > v_j \text{ for all } j \neq i \\ 0 & \text{otherwise} \end{array} \right\}$$

$$c_i(v_1, .., v_N) = \left\{ \begin{array}{ll} \hat{b}(v_i) & \text{if } v_i > v_j \text{ for all } j \neq i \\ 0 & \text{otherwise} \end{array} \right\}$$

   Easy to check: truth telling is an equilibrium of this direct mechanism.

2. Second-Price, English: Same assignment function as First Price and

$$c_i(v_1, .., v_N) = \left\{ \begin{array}{ll} \hat{b}(v_{\text{second}}) & \text{if } v_i > v_j \text{ for all } j \neq i \\ 0 & \text{otherwise} \end{array} \right\}$$

For $r_i \in [0,1]$, define:

$$\begin{aligned}\bar{p}_i(r_i) &= \int_0^1 .. \int_0^1 p_i(r_i, v_{-i}) f_{-i}(v_{-i})\, dv_{-i}; \\ \bar{c}_i(r_i) &= \int_0^1 .. \int_0^1 c_i(r_i, v_{-i}) f_{-i}(v_{-i})\, dv_{-i}.\end{aligned}$$

Then,

$$u_i(r_i, v_i) = \bar{p}_i(r_i)\, v_i - \bar{c}_i(r_i)\,.$$

A direct mechanism is incentive compatible if

$$u_i(v_i, v_i) \geq u_i(r_i, v_i) \quad \forall i \forall r_i \forall v_i.$$

Proposition 8 *A direct mechanism is incentive-compatible if and only if*

1. $\bar{p}_i(v_i)$ *is non-decreasing in* v_i.

2. $\bar{c}_i$ *can be written as*

$$\bar{c}_i(v_i) = \bar{c}_i(0) + \bar{p}_i(v_i)\, v_i - \int_0^{v_i} \bar{p}_i(x)\, dx.$$

Sketch of Proof. For (1), take $v' < v''$ and suppose that $\bar{p}_i(v') > \bar{p}_i(v'')$. Incentive compatibility implies:

$$\begin{aligned} u_i\left(v', v'\right) &\geq u_i\left(v'', v'\right); \\ u_i\left(v'', v''\right) &\geq u_i\left(v', v''\right). \end{aligned}$$

Sum the two inequalities

$$u_i\left(v', v'\right) + u_i\left(v'', v''\right) \geq u_i\left(v'', v'\right) + u_i\left(v', v''\right)$$

That is

$$\bar{p}_i\left(v'\right)v' + \bar{p}_i\left(v''\right)v'' \geq \bar{p}_i\left(v'\right)v'' + \bar{p}_i\left(v''\right)v'.$$

which re-writes as

$$\left(\bar{p}_i\left(v''\right) - \bar{p}_i\left(v'\right)\right)\left(v'' - v'\right) \geq 0,$$

which is a contradiction because $v' < v''$ and $\bar{p}_i(v') > \bar{p}_i(v'')$.

For (2), note that incentive compatibility implies the first-order condition

$$\frac{d}{dr} u_i(r, v)\Big|_{r=v} = 0 \quad \forall v.$$

We have

$$\frac{d}{dr} u_i(r,v) = \bar{p}_i'(r)\, v - \bar{c}_i'(r)$$

Hence

$$\bar{c}_i'(v) = \bar{p}_i'(v)\, v \quad \forall v.$$

Integrating both sides

$$\bar{c}_i(v) - \bar{c}_i(0) = \overbrace{\bar{p}_i(v)\, v}^{\text{expected benefit}} - \overbrace{\int_0^v \bar{p}_i(x)\, dx}^{\text{rent}},$$

which corresponds to (2). ■

To interpret (2), go back to mechanism design and think of downward local IC constraints:

$$\hat{t}_i = \hat{t}_{i-1} + u_1(\hat{x}_i, \theta_i) - u_1(\hat{x}_{i-1}, \theta_i) \quad \text{for } i = 1, ..., n$$

Hence

$$\begin{aligned} \hat{t}_i &= u_1(\hat{x}_i, \theta_i) - u_1(\hat{x}_{i-1}, \theta_i) \\ &\quad + u_1(\hat{x}_{i-1}, \theta_{i-1}) - u_1(\hat{x}_{i-2}, \theta_{i-1}) \\ &\quad + ... \\ &= u_1(\hat{x}_i, \theta_i) \\ &\quad - (u_1(\hat{x}_{i-1}, \theta_i) - u_1(\hat{x}_{i-1}, \theta_{i-1})) \\ &\quad - ... \\ &= u_1(\hat{x}_i, \theta_i) - \sum_{k=1}^{i-1} (u_1(\hat{x}_{i-k}, \theta_{i-k+1}) - u_1(\hat{x}_{i-k}, \theta_{i-k} \end{aligned}$$

Theorem 9 (Revenue Equivalence) *If two incentive-compatible direct selling mechanisms have the same probability assignment functions and every bidder with valuation zero is indifferent between the two mechanisms, then the two mechanisms generate the same expected revenue.*

Proof. As the v's are independent, the expected revenue can be written as

$$\begin{aligned} & R \\ = & \sum_{i=1}^{N} \int_0^1 \bar{c}_i(v_i) f_i(v_i)\, dv_i \\ = & \sum_{i=1}^{N} \int_0^1 \left(\bar{c}_i(0) + \bar{p}_i(v_i) v_i - \int_0^{v_i} \bar{p}_i(x)\, dx \right) f_i(v_i)\, dv_i \\ = & \sum_{i=1}^{N} \bar{c}_i(0) + \sum_{i=1}^{N} \int_0^1 \left(\bar{p}_i(v_i) v_i - \int_0^{v_i} \bar{p}_i(x)\, dx \right) f_i(v_i)\, d \end{aligned}$$

The revenue depends only on $\bar{c}_i(0)$ and $\bar{p}_i$, not on $\bar{c}_i(v_i)$. ■

We can now add new formats to the list. See exercise.

Efficiency

The four formats have the same p.

They allocate the object to the bidder with the highest valuation.

Efficient.

Trade-off between efficiency and revenue maximation:

- The auctioneer can increase expected revenue by setting a reserve price.

- A reserve price mis-allocates the object with positive probability.

Reserve Price in Second-Price Auction

The auctioneer uses a second-price auction with reserve price $r \geq 0$. If all the bids are below r, the auctioneer keeps the object (for which she has zero utility).

Proposition 10 *The optimal reserve price is strictly greater than zero.*

As before, $h(v)$ is the density function of the second-highest element v^{second} of $\{v_1, ..., v_N\}$.

The expected revenue is

$$\begin{aligned} R_2 &= \int_r^1 \hat{b}(v)\, h(v)\, dv + r \Pr\left(v^{\text{second}} < r, v^{\text{first}} > r\right) \\ &= N(N-1) \int_r^1 v\,(F(v))^{N-2} f(v)\,(1 - F(v))\, dv \\ &\quad + rN\,(F(r))^{N-1}\,(1 - F(r)) \end{aligned}$$

Take derivatives:

$$\begin{aligned}\frac{dR_2}{dr} &= -N(N-1)\,r\,(F(r))^{N-2} f(r)(1-F(r)) \\ &\quad + N(N-1)\,r\,(F(r))^{N-2} f(r)(1-F(r)) \\ &\quad - rN(N-1)(F(r))^{N-1} f(r) \\ &\quad + N(F(r))^{N-1}(1-F(r))\end{aligned}$$

and

$$\frac{\left.\frac{dR_2}{dr}\right|_r}{(F(r))^{N-1}} = -rN(N-1)f(r) + N(1-F(r))$$

Then

$$\lim_{r\to 0^+} \frac{\left.\frac{dR_2}{dr}\right|_r}{(F(r))^{N-1}} = N > 0$$

For r small enough, $\left.\frac{dR_2}{dr}\right|_r$ is positive and a higher r increases the auctioneer's expected revenue.

Common Values

So far, we have assumed that valuations are indepently distributed.

But think of auctions for

- Oil fields
- New issues of securities
- Spectrum (UMTS)
- Any object which could be re-sold (paintings, cars, etc).

Values are then interdependent.

Let us look at the most extreme case: the value is the same for every player (but still stochastic):

$$v_1 = ... = v_n = v$$

and v has density f and CDF F on $[0, 1]$

Buyer i observes signal y_i with distribution $g(y_i|v)$. Assume that the y's are independent across buyers conditional on v.

Restrict attention to second-price auctions.

Is the equilibrium of the game

$$b_i(y_i) = E[v|y_i]? \tag{4}$$

No. A buyer who bids $E[x|y_i]$ is paying too much on average.

To see this, suppose everybody bids according to the *naive* strategy in (4). If i wins, it means that

$$E[v|y_i] = \max(E[v|y_1], ..., E[v|y_n])$$

equivalent to

$$y_i = \mathsf{max}\left(y_1, ..., y_n\right)$$

But then

$$E\left[v|y_1, ..., y_n\right] < E\left[v|y_i\right]$$

If i had known what the others know he would have bid less. This is the *winner's curse*.

In equilibrium, rational bidders are not subject to the winner's curse because they do not use a naive strategy.

The equilibrium strategy is the *sophisticated* bid function:

$$\tilde{b}_i\left(y_i\right) = E\left[v|y_i, y_i = \max_{j \neq i} y_j\right],$$

i.e. a buyer conditions his bid on the event his bid is equal to the second-highest bid.

Are bidders rational? Experimental evidence (Kagel and Levin 1986): both naive and strategic bidding.

Information Provision and Revenue Maximization

Should the auctioneer allow bidders to get more information about the object for sale?

Example: provide an independent expert report.

Suppose the cost of information provision is zero

Milgrom-Weber (Econometrica 1982)

Theorem 11 *In symmetric environments, if the auctioneer uses a first- or second-price auction the best reporting policy is full disclosure.*

In our example:

Suppose the auctioneer chooses between: (1) letting bidders know only y_i; (2) providing them with perfect information (they learn v).

With (2), the bid is simply $b_i = v$ and each buyer gets zero expected payoff.

Which Format?

With common values, the Revenue Equivalence Theorem does not hold.

It is still true that First $=$ Dutch, but

Milgrom-Weber prove:

- English $>$ Second Price.

  Intuition: The sequential format provides more info.

- Second Price $>$ First Price (if bidders are risk-neutral):

  Intuition: reduce winner's curse